PHYSICAL REVIEW E **74**, 031905 (2006)

Internal signal transmission in one-way coupled excitable system: Noise and coupling effects

Qianshu Li* and Xiufeng Lang
The Institute for Chemical Physics, Beijing Institute of Technology, Beijing, China, 100081

(Received 22 November 2005; published 8 September 2006)

We study the spatiotemporal dynamics of a one-way coupled FitzHugh-Nagumo system of twenty neurons, which is subject to external noise at the first neuron. It is shown that noise-induced oscillation (NIO) triggered at the first neuron is propagated along the chain with noise suppression, such that a rather "regular" signal is obtained at the last neuron, which can provide a mechanism for the creation of the informative signal in a neural network. Coherence resonance or coherence biresonance behavior appears in the transmission of NIO at appropriate coupling and the information flow in each neuron can be simultaneously optimized at the optimal value of noise.

DOI: 10.1103/PhysRevE.74.031905 PACS number(s): 87.19.La, 05.40.−a, 87.10.+e

I. INTRODUCTION

In the phenomenon of stochastic resonance (SR), a small periodic signal forcing nonlinear system can be amplified by the addition of a stochastic force, or external noise, to the signal [1]. Even without a periodic signal, coherent oscillation in a nonlinear system can be induced purely by noise and its coherence displays resonantlike behavior with the variation of noise intensity [2,3]. The SR-like phenomenon is called autonomous SR [2,4,5], coherence resonance (CR) [3,6], or internal SR [7], which has been demonstrated in a variety of excitable systems [3], bistable systems [2,8,9], and systems with delay [10]. Coupling such stochastic resonators into an array enhances the SR or CR effect and produces richer spatiotemporal dynamical phenomena, including array-enhanced stochastic resonance [11], array-enhanced coherent resonance [12], spatiotemporal stochastic resonance [13], noise-induced pattern transition [14], and noise-enhanced propagation (NEP) [15].

It often happens that real systems are often modeled by an array of coupled dynamic elements subject to an external signal at only one end, such that the NEP behavior is of great importance, especially for signaling processes in living systems. In recent years, many investigations [16–19] have illustrated that noise can sustain and support the signal transmission through a linear array of one-way coupled oscillators, which are subject to an external signal or external noise at one end of the chain. For example, Zhang *et al.*[20] demonstrated that, with sufficient coupling, noise can induce an undamped signal transmission in an array of one-way coupled bistable elements. Postnov *et al.*[19] experimentally observed that the array of four-stage cascaded systems, excited by white noise at the first monovibrator, can form a regular output signal gradually along the chain.

How information is encoded and transmitted in the neurons has been studied in the field of neuroscience for several decades. Neural systems are the best candidates to investigate noise effects on information transmission since noise is ubiquitous, arising from the external input from the ambient noise and the internal input from other neurons. In the past decade, a variety of numeric and experimental studies has observed the SR or CR phenomena in neurons and confirmed that noise supports signal code or signal transmission in neurons [3,20]. In neural networks, most of the previous work[21] has reported the effects of noise and network topology on the collective response of coupled neurons to the external signal or external noise. In 2000, Rosa *et al.* [22] just studied the spatiotemporal dynamics of unidirectionally coupled Hindmarsh-Rose (HR) neurons. It is well established that the unidirectionally coupled neural model is motivated by biological problems related to the dynamics of central pattern generators [23], which are responsible for the control of every rhythmic activity in animals. Based on previous work in neural networks and the significance of a one-way coupled neural model, we investigate noise effects on the transmission of internal signal, arising from noise-induced oscillation (NIO), through a one-way coupled neural channel in the present paper. The results showed that the external noise injected to the first neuron was transferred into a regular output signal at the last neuron through the channel at appropriate noise and coupling, which implies that the one-way coupled neural network, excited by white noise, is able to provide the spatial regularity of the desired level.

II. DYNAMICAL MODELS AND METHODS

A. The original model and one-way coupled model

In Ref. [24], Chialvo *et al.* reported on the question of how information contained in a neural message is transmitted, depending on two schemes of encoding: stochastic or deterministic and found that there was an optimum noise amplitude for information transmission to minimize the information loss in a sensory neuron. Based on the findings, we investigated the effect of noise on internal signal transmission in coupled neurons. In order to make the investigation general, herein, we have chosen the FitzHugh-Nagumo (FHN) model, which is a simple but representative model of an excitable system and nerve pulses [3]. Originally the model was suggested for the description of nerve pulses [25] and its dynamical equations are as follows:

$$\varepsilon \frac{dx}{dt} = x - \frac{x^3}{3} - y, \tag{1}$$

---

[*] Corresponding author. Email address: qsli@bit.edu.cn

$$\frac{dy}{dt} = x + a, \tag{2}$$

where $\varepsilon = 0.01$ and a is the control parameter. The control parameter a governs the character of solutions: if $|a| > 1$, the system has only a stable fixed point, while $|a| < 1$ a limit cycle appears. For $|a|$ slightly larger than one, the system with fixed point dynamics is excitable because it will return to the fixed point only after a large excursion when perturbed away from the fixed point.

The one-way coupled identical FHN system could be described by the following equations:

$$\varepsilon \frac{dx_i}{dt} = x_i - \frac{x_i^3}{3} - y_i + g(x_{i-1} - x_i), \tag{3}$$

$$\frac{dy_i}{dt} = x_i + a_i. \tag{4}$$

Here i running from 2 to N is the index of the neurons ($N = 20$ is the total number of neurons), and g is the coupling strength. Herein, for our purpose, we suppose that the control parameter a is the same for the neurons, that is, we keep $a_i (i = 1, \ldots, N)$ identically to be $a_0 = 1.05$, which is slightly larger than one, such that no oscillation exists in the absence of noise. Then we perturb the control parameter a_1 of the first neuron, i.e., $a_1 = a_0 + D\xi(t)$, where D denotes the intensity of Guassian white noise $\xi(t)$ with zero mean value $\langle \xi(t) \rangle = 0$ and unit variance $\langle \xi(t)\xi(t+\tau) \rangle = \delta(\tau)$. Since the behavior of the first neuron ($i = 1$) is still described by Eq. (1) and the dynamic behavior of the first neuron is not affected by the coupling, one would expect to find NIO and CR, as has already been reported in Ref. [3], where CR is characterized by the appearance of a maximum in correlation time τ_c and a minimum in the normalized fluctuations of pulse durations R of noise-induced oscillation with the variation of the noise amplitude D. Note that as shown in Fig. 4(a), we have also found the CR behavior in the first neuron.

B. Methods

To describe the temporal coherence of the firings in a neuron quantitatively, we introduce a statistical measure, i.e., the characteristic correlation time. The normalized autocorrelation function $C(\tau)$ of the variable y and the corresponding characteristic correlation time τ_c are calculated as follows [3,26]:

$$C(\tau) = \frac{\langle \tilde{y}(t)\tilde{y}(t+\tau) \rangle}{\langle \tilde{y}^2 \rangle}, \quad \tilde{y} = y - \langle y \rangle \tag{5}$$

$$\tau_c = \int_0^\infty C^2(t)\,dt, \tag{6}$$

where $\langle \cdot \rangle$ denotes average over time. To further characterize and confirm the temporal coherence, we compute another quantity, that is, the normalized fluctuations of pulse durations (which can be interpreted, in the context of SR terminology, as noise-to-signal ratio). The normalized fluctuation of pulse durations R of the variable y is obtained by the following formula [3]:

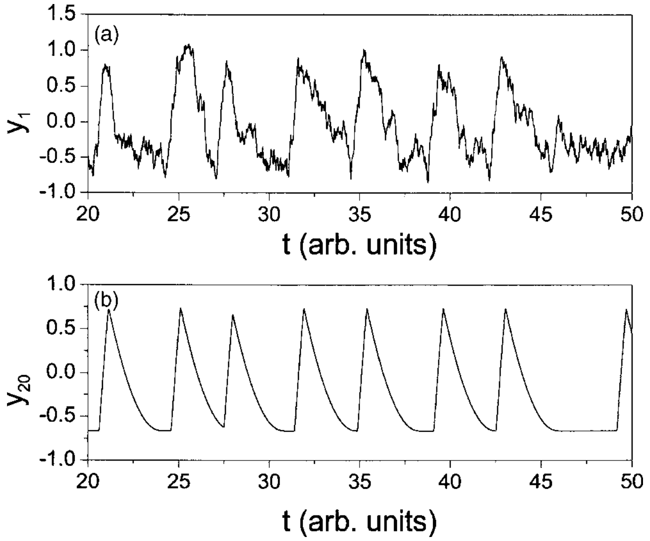

FIG. 1. The time series of y in the first neuron (a) and the twentieth neuron (b) with $D = 0.448$ and $g = 0.2$.

$$R = \frac{\sqrt{\mathrm{Var}(T_k)}}{\langle T_k \rangle}. \tag{7}$$

Here $T_k = t_{k+1} - t_k$, and t_k are the time of the kth firing of a neuron. Note that a pulse occurs when the state variable y exceeds a certain threshold value y_0 (here taken arbitrarily as $y_0 = 0.3$) and it turns out that the threshold value can vary in a wide range without altering the results. Both τ_c and R quantitatively characterize the regularity of the pulses: a larger value of τ_c and a smaller value of R mean more closeness of a pulse train to a periodic one, where R is obviously zero.

III. RESULTS AND DISCUSSION

We integrate the Eqs. (1)–(4) numerically using the Euler method with the time step of 0.001 for various noise intensities and coupling strengths. Note that the values of τ_c and R are calculated by averaging the results of 20 runs. When the dynamical behavior of the neurons is investigated one by one, it can be found that the NIO triggered at the first neuron has propagated along the chain with noise suppression, leading to a rather "regular" output signal at the end neuron. For example, the results obtained for $N = 20$, $D = 0.448$, and $g = 0.2$ are displayed in Fig. 1 and Fig. 2. Figures 1(a) and 1(b) display the time series of y_1 and y_{20}, respectively, which illustrate that the interspike interval of y_{20} is much more regular than y_1 and the amplitude of y_{20} is slightly smaller than that of y_1. When other neurons in the system are investigated, it is also found that the time series of y_i are much smoother than those of y_{i-1} and their amplitudes are nearly identical, close to that of y_1 (the data are not shown here). Recently, Xin *et al.* [18] have found that NIO triggered at the first oscillator can propagate along one-way coupled chemical oscillators with noise suppression and considerable am-

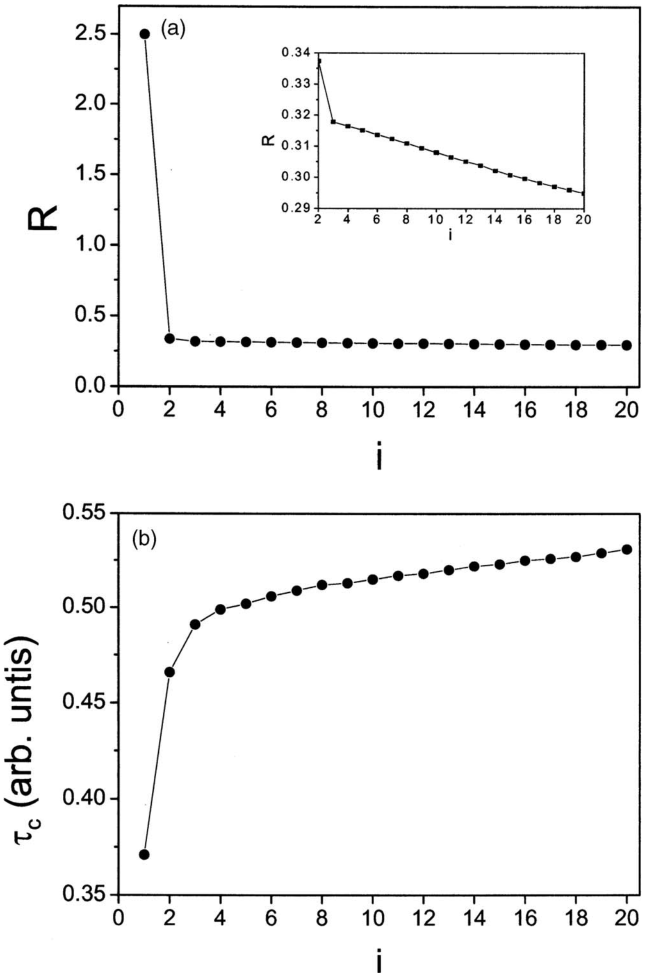

FIG. 2. (a) The noise-to-signal ratio R and (b) the correlation time τ_c versus the neuron number i. The parameters are the same as those in Fig. 1. The dependence of R on $i(i \geq 2)$ is shown in inset of Fig. 2(a).

plification. Herein, noise is gradually reduced along the one-way coupled neurons, and the amplitude of the firings in each neuron hardly changes, because the intrinsic dynamics of the neuron differs from that of the chemical oscillator. To further exhibit the phenomenon, τ_c and R of all neurons are calculated for $g=0.2$ and $D=0.448$, shown in Figs. 2(a) and 2(b), where the curves of τ_c and R both demonstrate the coherence of the firings is enhanced gradually along the one-way coupled FHN neurons, which is the same as the effect of regularity gain observed in Ref. [19]. As in the presentation in Ref. [27], this phenomenon may happen as follows: the firings of the first neuron induced by an external noise are more or less irregular, so that the second neuron inherits the more or less irregular firings. However, the second neuron shows a more periodic response to it through the assistance of the nonlinearity of the neuron. In the same way, the regularization of the firings sequentially occurs with the propagation of the internal signal from the (i-1)th neuron to the ith neuron. Therefore, although firings on the first neuron are purely induced by noise, the coherence of firings is enhanced by the cooperation between the coupling and nonlinearity of the neuron.

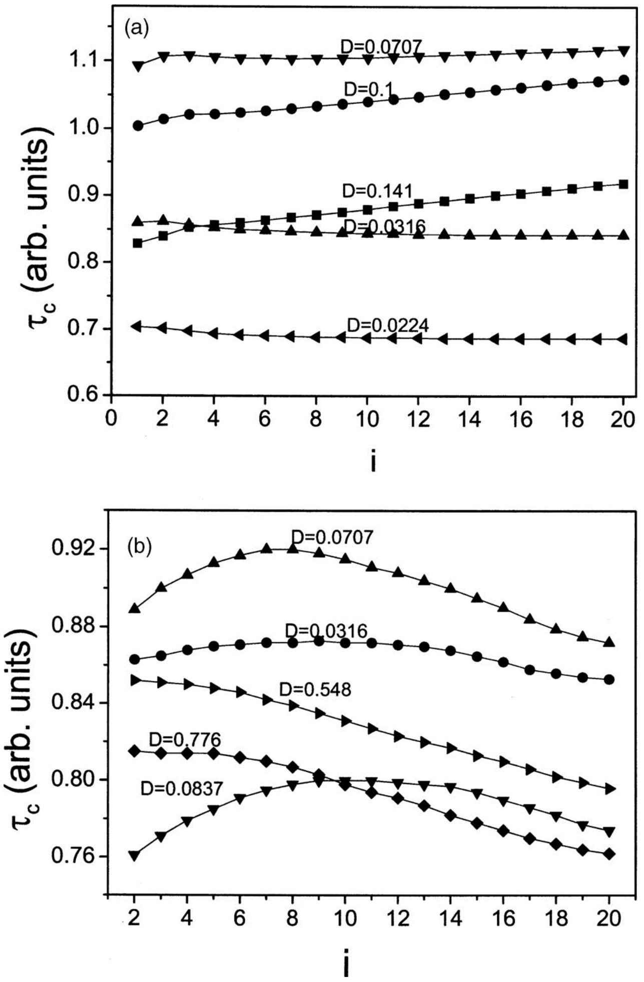

FIG. 3. The correlation time τ_c versus the neuron number i for various D by fixing (a) $g=0.1$ and (b) $g=0.01$. The noise intensities are labeled on each curve of the figure.

In the following, we study the effects of noise and coupling on the internal signal transmission properties, especially the regularity gain phenomenon. The dependence of τ_c on i for various noise intensities at $g=0.1$ and $g=0.01$ are portrayed in Figs. 3(a) and 3(b) respectively. In Fig. 3, two interesting features are clearly revealed: (i) There exists an optimal noise intensity for the most effective signal propagation, which is similar to the SR-like phenomenon of information measurement in Ref. [24]. Very low or high noise can deteriorate the coherence of the oscillation. (ii) At a high level of noise, the NIO triggered at the first neuron can propagate along the chain with enhancing coherence at strong coupling or decreasing coherence at weak coupling, which implies that the phenomenon of regularity gain can only appear at strong coupling and high noise. For example, Fig. 3 shows that the most effective propagation of the NIO appears at $D=0.0707$ for the two coupling strengths and for other noise intensity, the coherence of the firings in each neuron is destroyed greatly, consequently, decreasing the transduction efficiency of the signal in each neuron. Furthermore, it is clear that coherence of the firings is evidently enhanced along the long chain at $D=0.1$ and $D=0.141$ in Fig. 3(a), while significantly decreased at $D=0.548$ and D

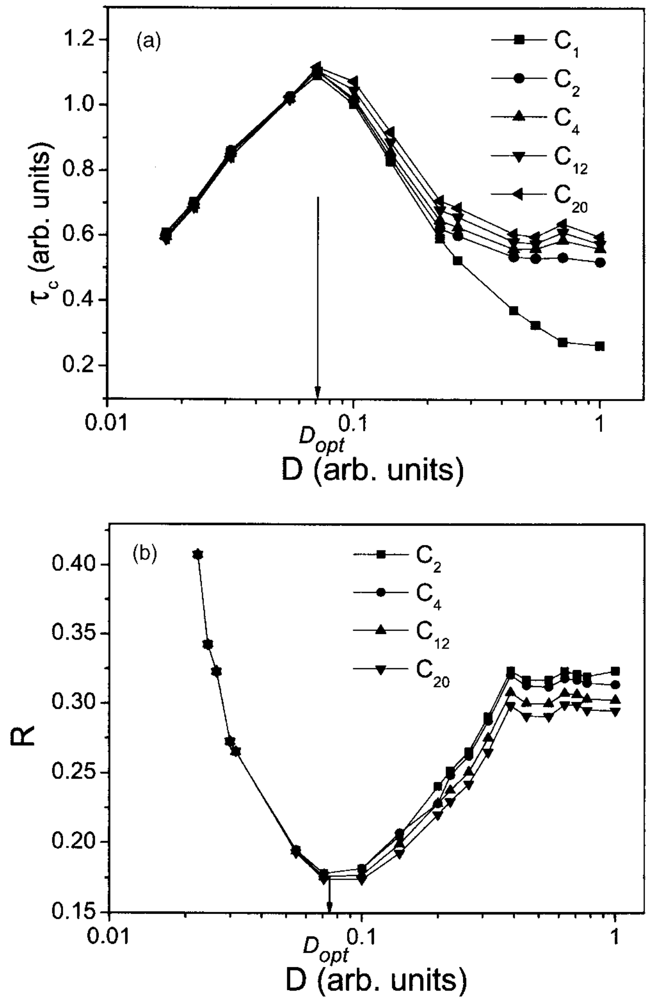

FIG. 4. (a) The correlation time τ_c and (b) the noise-to-signal ratio R versus the noise intensity D for the neurons with $g=0.1$. Note that C_i denotes the ith neuron of the coupled system and the curve of C_1 displays coherence resonance in the first neuron in the figure.

$=0.776$ in Fig. 3(b). Interestingly, Fig. 3(b) shows that there exists an optimal neuron in coupled neurons at noise intensity $D=0.0837$ or $D=0.0707$. Therefore, the signal in the two neurons is the most regular for the fixed noise intensity and coupling strength, which implies that for appropriate noise and coupling, the information flow can be optimized in a intermediate neuron of coupled neurons. Recently, some reports [28] have demonstrated that the collective behavior of an array of coupled noisy elements is the most ordered when the number of elements has an optimal value. The results obtained here illustrate that there exists the most ordered oscillation in a neuron of an array of coupled neurons.

The optimal noise intensity and the effect of regularity gain are manifested more clearly in Figs. 4 and 5. The curves of τ_c and R versus D in Fig. 4 show CR behaviors appearing at a same optimal noise intensity D_{opt}, which means that the efficiency of signal transduction in all neurons is optimized at a optimal value simultaneously. It is emphasized that similar phenomenon has been observed in Ref. [24]. Furthermore, at a high level of noise, the increase of τ_c and the decrease of R with the increment of i further validates that the spatial regularity of the firings is enhanced along the chain as illustrated in Fig. 3(a). Note that the data for $g=0.2$ and $g=1$ has been computed and similar phenomena have also been found. These results are consistent with the experimentally observed behavior: array-enhanced propagation [27]. Figures 5(a) and 5(b) display the dependence of τ_c and R on D for various i by fixing $g=0.01$, respectively. It is shown that the dynamical behaviors in the neurons for $g=0.01$ are obviously different from those for $g=0.1$. As shown in Fig. 5, coherence biresonance (CBR) behaviors clearly appear in the neurons except for the first neuron, where D_{opt1} is smaller and D_{opt2} is larger than D_{opt} in Fig. 4.

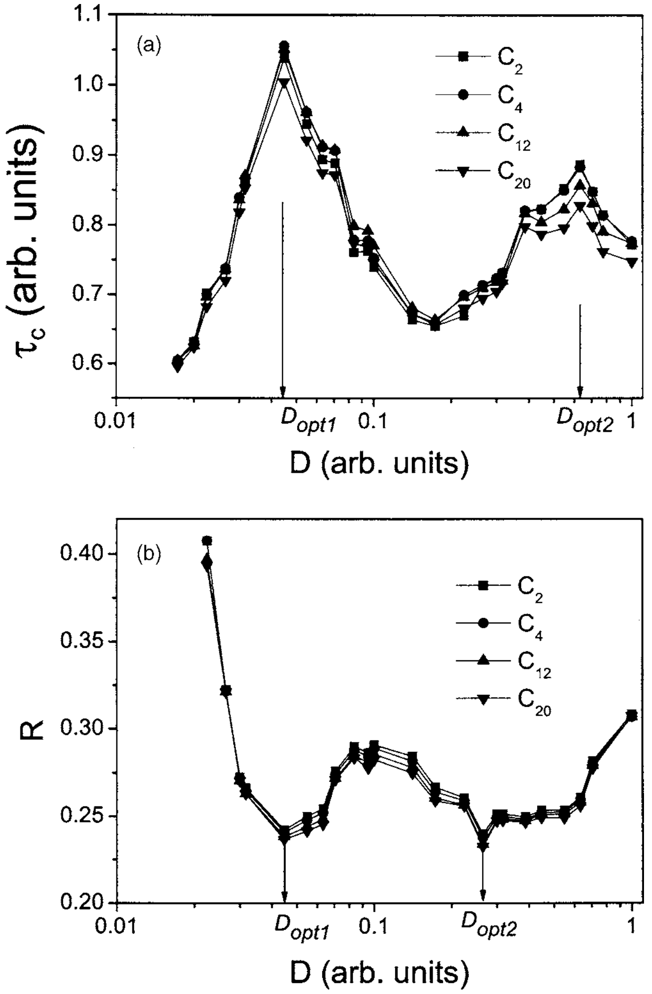

FIG. 5. (a) The correlation time τ_c and (b) the noise-to-signal ratio R versus the noise intensity D for the neurons with $g=0.01$. C_i denotes the ith neuron.

To further have a global view for the CR and CBR behavior, the contour plot of τ_c in the fourth neuron ($i=4$) for various D and g is shown in Fig. 6. It is clearly shown that the D-g plane is divided into two regions by a critical coupling strength $g=0.04$: The weak coupling region ($g<0.04$) and the strong coupling region ($g>0.04$). Coherence resonance behavior appears in the strong coupling region, while coherence biresonance behavior exists in the weak coupling region. It is noted that the data of τ_c in other neu-

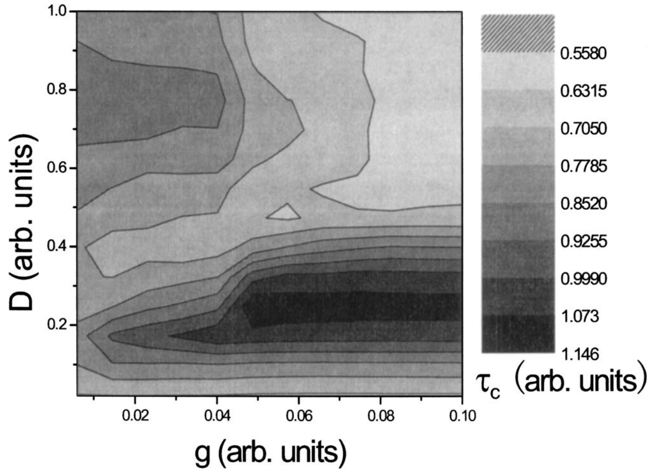

FIG. 6. Contour plot of the correlation time τ_c versus the noise intensity D versus the coupling strength g.

rons have also been calculated and revealed the same contours as that in Fig. 6. The values of τ_c at weak coupling are much larger than those at strong coupling for the high noise level, shown in Fig. 6, so the transduction of the internal signal in each neuron is more effective at the weak coupling than the strong coupling, such that weak coupling may be more favorable for the internal signal propagation through the channel at appropriate noise. It is emphasized that this is different from previous studies, where a certain coupling strength optimize the propagation of NIO [18] or stronger coupling was more favorable for noise-enhanced propagation of an external signal [15,16].

In the present paper, we found the effect of regularity gain in one-way coupled neurons, where each neuron is excitable and only the first neuron is subject to external noise. The information about stimulus intensity is encoded in the frequency of generated impulses for the neural network [29]. The effect of regularity gain can be considered one of the possible mechanisms for the creation of the information signal while the nerve impulses are transmitted along one-way coupled neurons [19]. Recently, it was demonstrated that sensory systems *in vivo* may work in the very vicinity of supercritical Hopf bifurcations through self-tuned mechanisms [30]. So the sensory neuron may respond to ambient input from adjacent neurons with regular electrical signals using the mechanism described herein. In addition, in Ref. [31], it is reported that among numerous internal sources of noise in a neuron, the most important one may be the synaptic input (or synaptic current) from other neurons via excitatory or inhibitory interaction, which is called "internal noise" and it has been demonstrated that "internal noise" assists the occurrence of SR in neuronal systems and recovers the experimental results of the nonzero SNR in the absence of the external noise [32]. Because the present system could model the response of a neuron to the synaptic input inherited from the previous neuron in neural system, in that sense our results can further validate the results obtained in Refs. [31,32], and illustrate that "internal noise" also assists the occurrence of CR in neuronal systems without the external noise. We hope that experimental observation of CR by "internal noise" is possible with standard equipment used in the studies of stochastic resonance [32].

IV. SUMMARY

In conclusion, we have demonstrated that local noise added to the first neuron could induce remote order in one-way coupled FHN neurons, all tuned to be excitable. Furthermore, the regularity of the output signal in a neuron is improved along the chain at strong coupling and high noise level. Coherence resonance and coherence biresonance appear in neurons at appropriate coupling strength and the most regular output exists in each neuron at the same optimum noise intensity. In addition, due to the CBR behavior, weak coupling could support internal signal propagation at a high level of noise. We anticipate that these results could yield insights into neural information processing and the obtained phenomena may provide a new understanding for the role of noise and coupling in sensory systems.

ACKNOWLEDGMENTS

The present work was supported by National Natural Science Foundation of China (Grant. No. 20433050). We are grateful to acknowledge Dr. Yaping Li and Lin Ji for helpful discussions.

[1] L. Gammaitoni, P. Hänggi, P. Jung, and F. Marchesoni, ChemPhysChem **70**, 223 (1998); P. Hänggi, ChemPhysChem **3**, 285 (2002).

[2] G. Hu, T. Ditzinger, C. Z. Ning, and H. Haken, Phys. Rev. Lett. **71**, 807 (1993).

[3] A. S. Pikovsky and J. Kurths, Phys. Rev. Lett. **78**, 775 (1997); B. Hu and C. S. Zhou, Phys. Rev. E **61**, R1001 (2000).

[4] L. F. Yang, Z. H. Hou, and H. W. Xin, J. Chem. Phys. **110**, 3591 (1999).

[5] A. Longtin, Phys. Rev. E **55**, 868 (1997).

[6] S. G. Lee, A. Neiman, and S. Kim, Phys. Rev. E **57**, 3292 (1998).

[7] Z. H. Hou and H. W. Xin, J. Chem. Phys. **111**, 1592 (1999).

[8] B. Lindner and L. Schimansky-Geier, Phys. Rev. E **61**, 6103 (2000).

[9] J. R. Pradines, G. V. Osipov, and J. J. Collins, Phys. Rev. E **60**, 6407 (1999).

[10] T. Ohira and Y. Sato, Phys. Rev. Lett. **82**, 2811 (1999).

[11] J. F. Lindner, B. K. Meadows, and W. L. Ditto, Phys. Rev. Lett. **75**, 3 (1995).

[12] C. Zhou, J. Kurths, and B. Hu, Phys. Rev. Lett. **87**, 098101 (2001).

[13] P. Jung and G. Mayer-Kress, Phys. Rev. Lett. **74**, 2130 (1995); F. Marchesoni, L. Gammaitoni, and A. R. Bulsara, *ibid.* **76**,

2609 (1996); M. Löcher, G. A. Johnson, and E. R. Hunt, *ibid.* **77**, 4698 (1996); J. M. G. Vilar and J. M. Rubi, *ibid.* **78**, 2886 (1997).

[14] Z. Hou, L. Yang, X. Zuo, and H. Xin, Phys. Rev. Lett. **81**, 2854 (1998).

[15] J. F. Lindner, S. Chandramouli, A. R. Bulsara, and M. Löcher, Phys. Rev. Lett. **81**, 5048 (1998).

[16] Y. Zhang, G. Hu, and L. Gammaitoni, Phys. Rev. E **58**, 2952 (1998).

[17] Y. Jiang and H. Xin, Phys. Rev. E **62**, 1846 (2000).

[18] Z. Hou, K. Qu, and H. Xin, ChemPhysChem **6**, 58 (2005).

[19] D. E. Postnov, S. K. Han, T. G. Yim, and O. V. Sosnovtseva, Phys. Rev. E **59**, R3791 (1999).

[20] J. K. Douglas, L. Wikens, E. Pantazelou, and F. Moss, Nature (London) **365**, 337 (1993); X. Pei, L. Wilkens, and F. Moss, J. Neurophysiol. **76**, 3002 (1996).

[21] L. F. Lago-Fernández, R. Huerta, F. Corbacho, and J. A. Sigüenza, Phys. Rev. Lett. **84**, 2758 (2000); R. Toral, C. R. Mirasso, and J. D. Gunton, Europhys. Lett. **61**, 162 (2003); C. Zhou, J. Kurths, and B. Hu, Phys. Rev. E **67**, 030101 (2003); O. Kwon, H. H. Jo, and H. T. Moon, *ibid.* **72**, 066121 (2005).

[22] M. L. Rosa, M. I. Rabinovich, R. Huerta, H. D. I. Abarbanel, and L. Fortuna, Phys. Lett. A **266**, 88 (2000).

[23] *Neurons, Networks and Motor Behavior* edited by P. S. G. Stein, S. Grillner, A. I. Selverston, and D. G. Stuart (MIT Press, Cambridge, 1997).

[24] D. R. Chialvo and A. V. Apkarian, Proc. SPIE **2036**, 57 (1993).

[25] A. C. Scott, Rev. Mod. Phys. **47**, 487 (1975).

[26] K. Miyakawa and H. Isikawa, Phys. Rev. E **64**, 046204 (2002).

[27] K. Miyakawa, T. Okano, and T. Tanaka, Phys. Rev. E **71**, 066202 (2005).

[28] A. Pikovsky, A. Zaikin, and M. A. de la Casa, Phys. Rev. Lett. **88**, 050601 (2002); H. Hong, B. J. Kim, and M. Y. Choi, Phys. Rev. E **67**, 046101 (2003).

[29] C. M. Gray, P. König, A. K. Engel, and W. Singer, Nature (London) **338**, 334 (1989).

[30] S. Camalet, T. Duke, F. Jülicher, and J. Prost, Proc. Natl. Acad. Sci. U.S.A. **97**, 3183 (2000); V. M. Eguiluz, M. Ospeck, Y. Choe, A. J. Hudspeth, and M. O. Magnasco, Phys. Rev. Lett. **84**, 5232 (2000).

[31] W. Wang and Z. D. Wang, Phys. Rev. E **55**, 7379 (1997).

[32] J. K. Douglass, L. Wilken, E. Pantazelou, and F. Moss, Nature (London) **365**, 337 (1993); K. Wiesenfeld and F. Moss, *ibid.* **373**, 33 (1995).